Notes

Introduction

1. Knight, F. (1921), *Risk, uncertainty and profit*, Beard Books, Washington, D.C., 2002, p. 19: “Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk. . . . The term ‘risk,’ as loosely used in everyday speech and in economic discussions, really covers two things which, functionally at least, in their causal relations to economic organization are categorically different. . . . The essential fact is that ‘risk’ means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. . . . It will appear that a measurable uncertainty, or ‘risk’ proper . . . is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term ‘uncertainty’ to cases of the non-quantitative type. It is ‘true’ uncertainty, and not risk . . ., which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition.”


4. Volatility is indeed volatile.


7. In hedge finance, current cash flows cover debt servicing (capital and interest). In speculative finance, they only cover interest payments. In Ponzi finance, they cover neither interest nor principal payments.

8. As measured by a Fisher volume index eliminating currency fluctuations.


collective memory, in Keynesianism vs monetarism and other essays in financial history, Allen and Unwin, London.

15. See, for example, Sargent, T. J. (1999), The conquest of American inflation, Princeton University Press, Princeton, NJ.

16. See appendix D.


1. The Progressive Emergence of Expectations


3. In 1720, according to Cantillon, the market interest rate in London rose from 5 percent to 60 percent per annum.


5. The closeness of Wicksell’s “gambling spirit” with Keynes’s “animal spirits” is striking.

7. And he adds the following: “Exactly the same effects would be visible with an unchanged, or even a higher, rate of interest, if meanwhile the expected profit on capital had considerably increased.”

8. Hicks summarized this argument by saying it assumes the “elasticity of expectations” with respect to actual price changes to be equal to 1.


17. Heterogeneity of forecasts, in modern parlance.


20. Cagan approached the problem in terms of forecasting error and error correction coefficients, while Allais put the stress on the decay of human memory.

2. Rational Expectations Are Endogenous to and Abide by “the” Model


3. In chapter 3, we shall see that both Cagan and Allais empirically encountered and recognized the shortcomings of exponential smoothing in the early 1950s. It is this empirical encounter that led Allais to the HRL formulation.


10. Knight would have said *risky* instead of *uncertain*.


12. Read *risk*.

13. Read *risk*.


20. In other words, \( P(E) = \sum_i P(E|H_i) \times P(H_i) \) for all \( i \).


26. Another suggestion is to do rolling recursive OLS linear regression on an arbitrarily decided number of observations.
39. The so-called Duhem-Quine problem.

**Introduction to Part II**


**3. Macrofoundations of Monetary Dynamics**

2. In *Value and capital*, Hicks uses the same starting point, but—as far as I can judge—he did not exploit it as far as Allais did.
3. One simply needs to consider the government as an additional business selling services against taxes and to present the rest of the world as another additional business buying (importing) domestic goods and services or selling (exporting) foreign goods and services.
6. Albeit arguably, fixed-term deposits are more likely to be precautionary balances than currency and demand deposits.
11. In the Innsbruck paper, the two functions depend only on \( x = \frac{1}{D} \frac{dD}{dt} \), the latest growth rate in nominal spending. In the two later papers, they depend on the exponentially smoothed sequence of growth rates.

12. This function was chosen for its simplicity and illustrative properties. It belongs to this book, not to Allais, but, needless to say, it is directly inspired by his later works.

4. Microfoundations of Monetary Dynamics


2. The two formulations become equivalent when the rate of inflation becomes dominant relative to the real rate of growth.

3. As in the Uppsala (1954) and Paris (1955) papers, Allais observes that this formulation is very close to Boltzmann’s oblivion function and to Volterra’s dampening function.

4. This recursive relationship makes it very easy to compute an exponential average.


7. In physical time.

8. In psychological time.

9. In physical time.

10. In psychological time.

11. Besides Einstein, there are many examples in literature and politics of implicit reference to psychological time. For example, Lenin is believed to have said, “Sometimes decades pass and nothing happens, and then sometimes weeks pass and decades happen.”

12. The term \( Z \) is nothing but the numerator of relationship 4.2 under the assumption that \( r \) varies over time.

13. One of the fundamental principles of economic analysis is to present economic issues as optimization problems in which a certain quantity must be minimized or maximized. According to this principle, it seems logical to conjecture that relationship 4-31 (or 4-52) should be the solution of a certain optimization problem that, as it happened, Allais has not explicitly laid out. I wish I had been able to formulate this important problem before this book goes to press. Further research will hopefully close this gap.

14. For \( E \approx 0, e^{-aE - \gamma} \approx (1 - \alpha E) e^{-\gamma} \).

16. If $\alpha = 0$ or $b = 0$, then $\Psi(Z) = 1$ and the HRL formulation becomes equivalent to an exponential average, where $\chi_0$ remains the only parameter.


5. The Fundamental Equation of Monetary Dynamics


2. In the Anglo-Saxon literature, transactions are usually denoted by the letter $T$; however, as Allais uses the letter $T$ to denote his response period, it seems appropriate to designate transactions with the letter $Q$ to prevent any risk of confusion.

3. As opposed to the income velocity of money $v$, which is defined by the ratio $v = Y/M$, where $Y$ represents national income.

4. Leaving aside, for the time being, nonbank credit.


6. He even states that a first-rate business should show flexibility in this regard.


8. Samuelson’s oscillator has the same mathematical form, but its theoretical foundations, variables, and parameters are totally different.


6. Joint Testing of the HRL Formulation of the Demand for Money and of the Fundamental Equation of Monetary Dynamics

3. See, for example, Quantitative Micro Software, (2007) *EViews 6, User’s guide*, Irvine, CA or the works of the Nobel laureates Software who ‘formulated these techniques: Engle’, Granger, Sims.
4. For example, the GARCH(1,1) specification

\[ Y_t = X_t' + \epsilon_t \]  
\[ \sigma_t = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]

is equivalent to exponentially weighted moving average (EWMA) variance measures defined by the recursion

\[ \sigma_t^2 = (1 - \lambda)\epsilon_t^2 + \lambda \sigma_{t-1}^2 \]

where \( \omega = 0, \alpha = 1 - \lambda, \) and \( \beta = \lambda. \)


Notes 357


4. Allais used an IBM 7094 computer.


7. Case in which the constant is forced to zero, that is, when the average forecasting error is assumed to be equal to zero.

8. See section 2.12.


10. In his Economics of inflation, a book whose subject is the German hyperinflation, Bresciani-Turoni speaks most of the time of the depreciation of the mark. His original title in Italian is Le vicende del Marco Tedesco. The first sentence of Lionel Robbins’s foreword to the English edition starts with the words “The depreciation of the mark.”


13. This is in sharp contrast with Andrei Shleifer’s formulation where the reaction of positive feedback traders is supposed to be proportional to the latest price change. See Shleifer, A. (2000), Inefficient markets (An introduction to behavioral finance), Oxford University Press, Oxford.

8. The HRL Formulation and Nominal Interest Rates


2. During a conversation on April 25, 2002, Maurice Allais bluntly told the author of this book that his theory of the psychological rate of interest “did not work.”


6. Datastream time series: USGDP ... B.


1. Speculative or Ponzi finance to borrow Minsky’s words.
3. Datastream time series USCBDMGNA.
4. NBER Macro History database time series M14074: US loans on securities by member banks.
5. NBER Macro History database time series M12017a: Bank debits NYC USD bn.
6. NBER Macro History database time series M13003: NYSE 90-day time loan.
7. See for example the famous statement made by Chuck Prince, then Citigroup’s CEO, in an interview to the Financial Times on July 9, 2007: “When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you’ve got to get up and dance. We’re still dancing”.

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8. See table 4.5, page 89.


2. As 1 ducat weighs about 0.11 troy ounce, assuming a gold price of USD 1,300, one ducat is worth about 143 current USD in early 2014.
5. In many problems, ordinal utility suffices to find solutions.
9. Questions 35 and 36, the only two questions reproduced in Allais’s ubiquitously quoted 1953 Econometrica article.
10. 100 million 1952 FFR is roughly equivalent to 2.536 million 2012 USD.
11. Using 1 million as unit.
12. Without referring to utility, it is possible to give a less rigorous but may be more intuitive explanation of the Allais paradox. Respondents should have preferred $B$ to $A$ and $D$ to $C$, since $B$ and $D$ have higher mathematical expectation than, respectively, $A$ and $C$. Yet, in choice 1, a majority of respondents preferred $A$ to $B$, while in choice 2, a majority of the same respondents preferred $D$ to $C$. The preference $D ≻ C$ is consistent with the maximization of
mathematical expectation, since $D$ is the prospect with the higher probability-weighted gain (500m versus 110m). In contrast, the preference $A \succ B$ contradicts the maximization of mathematical expectation, since $A$ has lower mathematical expectation than $B$ (100m versus 139m). In terms of mathematical expectation, the difference between $A$ and $C$ is the same as between $B$ and $D$, namely, $89m = 89\% \times 100m$. The empirical observation that $A$ is preferred to $B$ and $C$ to $D$ suggests that the outcome (100m; 89%), which is embedded in both $A$ and $B$, is not taken into account independently from the other outcomes $(x_i, p_i)$ present in these two prospects, contrary to the independence axiom. In other words, the outcome (100m; 89%) seems to be valued more in $A$, where there is no risk at all, than in $B$, which entails some risk.


14. It is not clear whether prospect theory considers gains and losses in absolute or relative terms, although the first hypothesis seems the most likely.


16. See problems $P_3$ in table 10.4 and $P'_3$ in table 10.5.


19. In the first two problems, the values of the gains and losses are close enough to ignore the potential impact of utility. This approach is more questionable in the last two problems, even though local linearity of cardinal utility may still be assumed.

20. The *possibility effect* alluded to by Kahneman and Tversky can seemingly be measured by odd-order moments.

21. In these two problems, the gains are so close to each other that the potential impact of utility can safely be ignored.

22. Here, too, the potential impact of utility can probably be ignored as the values of the gains are in a maximum ratio of 2 to 1. The same remark applies to table 10.5.

23. The *certainty effect* alluded to by Kahneman and Tversky can seemingly be measured by even-order moments.


26. Before we proceed further, let us observe that an empirical observation of the general type

\[ \bar{s}(U_4) - \bar{s}(U_3) = \bar{s}(U_2) - \bar{s}(U_1) \]  

implies that cardinal utility can only be determined up to a linear transformation. Let us assume that cardinal utility is actually a linear transformation \( f \) of \( \bar{s} \), such that

\[ f(U) = c_1 \bar{s}(U) + c_2 \]  

Then if empirical observation reveals that relative to \( U_3 \), the gain \( U_4 \) provides the same increase in satisfaction as \( U_2 \) with respect to \( U_1 \), the only relationship we can write is

\[ f(U_4) - f(U_3) = f(U_2) - f(U_1) \]  

which is equivalent to

\[ c_1 \bar{s}(U_4) + c_2 - c_1 \bar{s}(U_3) - c_2 = c_1 \bar{s}(U_2) + c_2 - c_1 \bar{s}(U_1) - c_2 \]  

and finally to

\[ \bar{s}(U_4) - \bar{s}(U_3) = \bar{s}(U_2) - \bar{s}(U_1) \]  

The information contained in \( c_1 \) and \( c_2 \) is lost.

27. The answers given by the subject (Finetti in this case) are underlined; in his questionnaire, Allais formulated his questions using round numbers of 1952 French francs, such as 10, 25, and 50 million. To facilitate a contemporary reader’s introspection, these round numbers have been converted to 2012 USD and rounded to the nearest $1,000, at the risk of some persistent oddity.

28. In contrast, Kahneman and Tversky (1979) only allude to minimum perceptible thresholds.


31. \( d^2 \bar{s} / dU^2 < 0 \) and \( d^3 \bar{s} / dU^3 < 0 \)

32. In a lin-log graph, one plots the abscissa along a base-10 logarithmic scale and the ordinates along a linear scale.

33. Bear in mind that this analytical work was conducted some years before the advent of Excel, MATLAB, or Eviews as we know them today.

35. Negligible as it may seem, this small hump provides some interesting insights as regards hyperbolic discounting (see appendix E on intertemporal choice).


37. Questions 71 to 78 and 90 to 98.

38. 1952 French francs.

39. And not below.


1. From a purely formal point of view, there is no difference at all between mathematical expectation and a cocktail recipe. But someone ordering a gin and tonic is not expecting to be served two full glasses of gin first, followed by five full glasses of tonic (or vice versa).

2. To take an extreme example, until Monday, October 19, 1987, close of business, nobody knew that the Dow Jones Index could fall by 22.6 percent in one trading session.


4. By relative spread, we mean the difference between the logarithms of BAA and AAA bond yields.

5. Median price of existing homes.

6. A simple way to visualize the role of $\alpha$ is to observe the behavior of the logarithm of the rate of memory decay when the present value of past returns tends toward infinity. From the definition of $\chi(Z)$, we have
\[
\lim_{Z \to +\infty} \ln \chi(Z) = \alpha Z + \ln \chi_0 + \ln b - \ln(1 + b), \text{ which is the equation of an asymptotic line having } \alpha \text{ for slope.}
\]

7. With
\[
\chi_b(Z) = \frac{\chi_0 + b e^{\alpha Z}}{1 + b} \quad (1)
\]
and
\[
\chi_1(Z) = \frac{\chi_0 + b e^{\alpha Z}}{2} \quad (2)
\]
we have indeed
\[
\chi_b(Z) = \chi_1(Z) = \frac{\chi_0}{2} (e^{\alpha Z} - 1)(b - 1) \quad (3)
\]

8. Keynes’s famous analogy between financial markets and beauty contests could be given a quantitative content by assuming \( \alpha > 1 \) and \( b < 1 \), or \( \alpha' > 1 \) and \( b' > 1 \) or both!

9. This assumption concurs with recent research. See, for example, Malmendier, U., and Nagel, S. (2009), *Depression babies: Do macroeconomic experiences affect risk-taking?* and (2011), *Learning from inflation experience*, UC Berkeley and Stanford University, NBER and CPER. The two authors claim to have found that “individuals learn from data experienced over their life-times, rather than from all ‘available’ data.” For example, “young individuals place more weight on recently experienced inflation than older individuals,” or “individuals who have experienced low stock-market returns throughout their lives report lower willingness to take financial risk” and “recent return experiences have stronger effects, but experiences early in life still have significant influence, even several decades later.”

12. Conclusion

Appendix C Proofs

3. The subscript $e$ designates the equilibrium value of all the variables considered in this demonstration.
4. Neglecting second-order terms, we have, for example,
   \[
   \lim_{g \to 0} \frac{\Psi(Z_e + g) - \Psi(Z_e)}{g} = \Psi'(Z_e) \Rightarrow \Psi(Z) = \Psi(Z_e + g) \approx \Psi(Z_e) + g\Psi'(Z_e)
   \]
   \[
   \Psi(Z) \approx \Psi(Z_e) + g\Psi'(Z_e)
   \]  
   \[
   \frac{\Psi(Z)}{\Psi(Z_e)} \approx 1 + \frac{\Psi'(Z_e)}{\Psi(Z_e)} g \approx 1 - K_e g
   \]
   \[
   \frac{\Psi(Z)}{\Psi(Z_e)} \approx 1 - K_e g
   \]
   \[
   V(t) = V_e(1 + f(t)) \Rightarrow \ln V(t) = \ln V_e + \ln(1 + f(t))
   \]
   \[
   \frac{1}{V} \frac{dV}{dt} = \frac{d\ln V(t)}{dt} = \frac{d\ln(1 + f(t))}{dt} \approx \frac{df(t)}{dt}
   \]
   \[
   \frac{1}{V} \frac{dV}{dt} \approx \frac{df(t)}{dt}
   \]

Appendix E A Note on the Theory of Intertemporal Choice

4. Discount factors are the present value–future value ratios of outcomes deemed equivalent albeit distant in time.
5. See Table 6-1 in Frederick, S., Loewenstein, G., and O’Donoghue, T., *Time discounting and time preference*.
6. See chapter 8 of this book.
7. See Frederick, S., Loewenstein, G., and O’Donoghue, T., *Time discounting and time preference*.
8. A possibility alluded to by Daniel Bernoulli in 1738, quoting a contribution by Gabriel Cramer, dated 1728.
10. Please note that these rates are annual continuous (log) rates.
12. Relationship E.7 is identical to relationship 4.37, section 4.3, p. 76.