Introduction

MORE THAN any of the previous crises, through its sheer size, intensity, and costs, the global financial crisis that seemingly started in 2007 has challenged economists and their analytical tools. In a way, it is fair to say that this crisis has been and still is, first and foremost, a crisis in economic theory. Some accepted theories or paradigms, such as rational expectations and efficient markets, seem to have been radically wrong-footed by unnerving facts and pervasive uncertainty.1

This is somewhat of a paradox that we should be disappointed by the rational expectations hypothesis (REH), limited as its ambition ever was. Lucas himself, one of its most prominent proponents, wrote that “this REH will not be applicable in situations . . . which Knight called uncertainty . . . in cases of uncertainty, economic reasoning will be of no value.”2

But, for seeming lack of credible alternatives in a context of uncertainty (see table 1, p. xl), the economic profession as a whole seems reluctant to declare this paradigm obsolete. So strong is the hegemony of rational expectations that there are not many prominent economists who dare to declare the REH a dead end and who call loudly for an alternative theory of expectations formation. Phelps and Frydman are two of them. This is all the more notable that, through his contribution to the debate on the inflation-unemployment trade-off (the so-called Philips curve) in the 1960s, Phelps has been instrumental in pushing expectations to the core of macroeconomic analysis, while Frydman has been an early critic of rational expectations on both epistemological and empirical grounds.
In the introduction to a book, which they have recently co-edited, Phelps and Frydman have defined a research program for an alternative theory of expectations, a theory that would portray expectations as inputs of macroeconomic models instead of being an output of such models, as is the case under the rational expectations hypothesis.³

According to Phelps and Frydman, such a theory should acknowledge that

- Neither economists nor market participants have a complete understanding of how the economy works, because the world in which we live is one of uncertainty (“nonroutine change” in Phelps and Frydman’s vocabulary).
- Market participants are aware that the decisions made by others do have an impact on aggregate outcomes (interdependence).
- Economic and financial variables tend to switch back and forth between different regimes of low or high volatility (nonstationarity of data and heteroskedasticity⁴).

This alternative theory of expectations should do the following:

- Rest upon clearly specified microeconomic foundations.
- Not allow for systematic forecasting errors (for these would necessarily be arbitraged away).
- Derive expectations from a mathematical formalization.
- Not be mechanical (contrary to the standard adaptive expectations model).
- Be open to and able to deal with “nonroutine change,” but its time-varying structure should not be of a Markov-switching type; it should depend on context-dependent regularities.
- Reflect the empirical observation that market participants tend to revise their forecasts in “guardedly moderate ways” (bounded elasticity of expectations).
- Be able to deal with market participants’ heterogeneity.
- Be empirically testable.
- Describe the market participants’ response to their expectations.
- Offer insights on financial markets and financial instability.

Odd as it may seem, the main claim made in this book is that, already a long time ago, since it was in 1965, Allais has gone a very long
way in constructing a theory—his theory of psychological time and of
time-varying rate of memory decay—that meets all the requirements of
Phelps and Frydman’s research program. It goes even beyond, since it
very much relies on nonlinear relationships, a requirement enunciated
in another book, co-edited this time by Guesnerie.5

Economists who are inclined to distance themselves from the REH
have always found reasons to do so in financial markets. By their very
function, which is to trade claims on future income flows, financial mar-
kets are indeed a chosen country for controversies about the nature of
expectations. Guesnerie, for example, states that “it is not obvious, to
say the least, to explain actual stock markets’ fluctuations using dynamic
models that adopt some (not too loose) version of the REH.” As for
Phelps and Frydman, they assert in a much less understated tone that
“nowhere have REH’s epistemological flaws and empirical disappoint-
ments been more apparent than in efforts to model financial market
outcomes.” In his famous Manias, Panics and Crashes, Kindleberger
flatly rejects the idea that financial markets are rational and emphasizes
the role of mimicking behavior among market participants.

The Kindleberger model of manias, panics, and crashes starts with a
positive displacement: something happens—a technological innovation,
a change in regulation—that legitimately and rightfully raises profit ex-
pectations.6 As profit expectations rise, so does the demand for credit,
which is being accommodated by banks, nonbanks, or both. The early
successes draw new participants into the game. As a result, financing
structures become increasingly unstable, moving from hedge to Ponzi
finance, to borrow Minsky’s taxonomy of financing structures.7 It is
hard to conceive such a cumulative process without adaptive expecta-
tions playing a significant role, a bigger one at least than rational ex-
pectations. Then something else happens, not very well specified to say
the least, that causes the tide to reverse and pushes the late entrants
into insolvency.

Yet, it is difficult to explain some recent episodes of financial history
with the Kindleberger model. There are just too many instances where
it is difficult to identify a positive displacement prior to the start of the
rise in prices. Similarly, there is no obvious source of credit to explain
the continuation of the initial upward move. An interesting counterex-
ample is the US equity market between 1995 and 2000.

With the benefit of hindsight, the rise in real US equity prices from
1982 to the end of 1994 appears to have been a reversion to the secular
upward trend. On the basis of cyclically adjusted PEs, this 12-year rally brought valuations back to the record levels of the mid-1960s. Up to that point, it seems inappropriate to speak of a US equity bubble. But from early 1995 onward, things changed dramatically. The upward movement of prices kept on accelerating until April 1999 for the S&P 500 and March 2000 for the NASDAQ. This is when cyclically adjusted valuations entered unchartered territory. If there was a US equity bubble, it was between early 1995 and early 2000. Interestingly, neither the US nor the global money supply heralded the acceleration of the rise in equity prices. At best, it has been coincident. From the end of 1989 to the end of 1994, the annual rate of growth in US M2 and M3 slowed down from 5 percent to virtually zero. As for global money, its growth slowed down from 12 percent to 6.6 percent. Contrary to the US money supply, global money growth did not accelerate between 1994 and 1999. Finally, there was no noticeable change in the growth of nonbank credit in the United States prior to 1994. From all this, it seems difficult to conclude that credit and money started the equity bubble. As for positive displacement, it is only three to four years later that the Internet became an investment theme.

The Japanese equity market provides another interesting counterexample. The switch to bubble mode happened in late 1982 and early 1983, at a time when M2 annual growth was stabilizing between 7 percent and 8 percent, but definitely not accelerating. It is only four to five years later that money growth accelerated and that an alleged Japanese technological leadership became an investment theme.

The laboratory experiments conducted by Vernon L. Smith et al. confirm the shortcomings of the Minsky-Kindleberger model. In these experiments, a group of subjects is invited to trade an asset; contrary to markets for consumer goods, the prospect of resale does exist. An entire experiment consists of a sequence of 15 trading periods lasting altogether approximately 2 hours. In accordance with the REH, each subject is given complete information on the fundamental value of the asset, that is, on the statistical distribution of potential dividends. Furthermore, each subject is regularly reminded of this fundamental value, which converges toward zero as time goes by. Yet, time and again, for different groups of more or less sophisticated traders, bubbles occur in these laboratory markets: one can observe large price deviations from the fundamental value, until the market crashes during the last trading periods. The availability of credit does magnify these bubbles.
In a REH environment, such bubbles and crashes should never occur. Smith explains this paradox by the presence of momentum traders alongside rational value-driven investors. While the latter buy (or sell) when prices fall (or rise) below (or above) fundamental value, momentum traders get “into the market when prices are rising because [they] believe that [they’ll] be able to sell later at a higher price.” A seemingly more accurate definition would be that a momentum trader is someone who gets into the market when prices have been rising enough to convince him that he’ll be able to sell later at a higher price. Clearly keen not to utter the words “adaptive expectations,” Smith nevertheless uses this very concept pervasively. Later in this book, it will be shown that, notations aside, Smith’s mathematical formulation of the sentiment of momentum traders (“the price trend”) is identical to Cagan’s formulation of adaptive inflation expectations in 1954. Content with this explanation of bubble dynamics, Smith nevertheless concedes that “the sparks that ignite [bubbles], and the myopic, self-reinforcing behavioral mechanisms that sustain them, remain unpredictable [or mysteries].”

Time and again, too, Smith’s laboratory experiments reveal that “once a group of subjects experiences a bubble and crash over two experiments, and then return for a third experiment, trading departs little from fundamental value.” Defeated as it may be in the short run, rationality still seems to prevail in the long term. Or so Smith argues. Let us breathe a sigh of relief! Without strong conviction, Smith alludes to Renshaw’s hypothesis that “the severity of price bubbles and crashes in the economy is related to inexperience.”10 As time passes, new investors enter the market, old investors exit, and the proportion of investors remembering the last stock market decline changes. In 1896, Pareto had already formulated the same hypothesis.

Tempting as it is to reject or at least to amend the rational expectations hypothesis, the fact is that both the Minsky-Kindleberger model and Smith’s model have some shortcomings, too. Among other things, this book purports to show that the perceived risk of loss, defined as the present value of past drawdowns, may be the “spark” that ignites bubbles when it is low, as well as the safeguard against repeated bubbles when it is high. As a matter of fact, Smith’s experiments inadvertently initialize the perceived risk of loss at zero at the beginning of the first sequence of trading sessions.

The first objective of this book is to show that Allais’s11 little-known theory of monetary dynamics contains assumptions and analytical tools
that are—mutatis mutandis—liable to be easily transposed to financial markets and may contribute to modeling financial behavior. The most seminal of these tools is the hereditary, relativist, and logistic (HRL) formulation of the demand for money. The HRL formulation is indeed a model purporting to describe both how economic agents learn from their experience under the assumption of uncertainty and how they behave on the basis of this knowledge. This formulation contains a theory of psychological time and of time-varying rate of memory decay. Allais firmly believed “expectations” to be rooted in memory. Therefore, he was keen not to use the word “expectations” in his work on monetary dynamics. Yet, there is little doubt that Allais’s HRL formulation deals with what Fisher, Wicksell, Knight, Hayek, Keynes, and Hicks progressively identified as the issue of expectations. Milton Friedman made this point rather emphatically:12

“This work [the HRL formulation]13 introduces a very basic and important distinction between psychological time and chronological time. It is one of the most important and original paper that has been written for a long time... for its consideration of the problem of the formation of expectations.”

Allais’s HRL formulation can be interpreted as a general theory of expectations under uncertainty. It puts numbers on a range of concepts, which play a central role, albeit a purely literary one, in the discussion of endogenous financial instability, along the directions outlined by Irving Fisher, Hyman P. Minsky, and Charles P. Kindleberger.14

At the risk of a blatant anachronism, it would unexpectedly highlight the HRL formulation’s modernity to introduce it with the terminology that is dear to rational expectations theorists:15 one can indeed say that the HRL algorithm applies Bayesian inference through a nonlinear filter, the gain of which is time-varying since it is a nonlinear function of the latest estimate of the hidden state variable.16

In Cagan’s well-known adaptive expectations model, the elasticity of inflation expectations with respect to actual inflation is constant, and so is the elasticity of real money balances with respect to expected inflation.17 In Allais’s HRL formulation, the elasticity of expectations varies dynamically between almost 0, in a deflationary environment, and 1 during hyperinflations. As for the demand for money, it is a monotonic, decreasing, bounded nonlinear function, the elasticity of which with respect to “expected” inflation varies between almost 0, in deflationary conditions, and −1, when hyperinflation is rife.
Another of Allais’s seminal contributions is his fundamental equation of monetary dynamics (FEMD). This equation describes how the gap between outstanding and desired money balances influences relative changes in the transactions velocity of money and hence in aggregate nominal spending. The embedding of the HRL formulation into the fundamental equation of monetary dynamics generates different types of equilibrium (stable or unstable) as well as various types of convergence paths toward equilibrium (pseudo-periodic or aperiodic), depending on the values taken by certain parameters. Within this framework, even when the supply of money is constant or hardly grows, it is possible, at least temporarily, to generate convex paths of accelerating growth rates. According to Cagan, while theoretically conceivable, such an outcome has never been observed in the real world. Vernon Smith has observed it in laboratory experiments of asset markets. Furthermore, when the supply of money grows, the HRL model of monetary dynamics can generate paths that mimic Vernon Smith’s “echo bubbles.”

The second objective of this book is to show that Allais’s well-known paradox can be unified with his theory of psychological time and time-varying rate of memory decay into the concept of perceived risk of loss, that is, the present value of the sequence of drawdowns experienced in a given market. In an uncertain world à la Knight, Keynes, and Minsky, the perceived risk of loss is an alternative not only to both rational and standard adaptive expectations models, but also to the variance of returns as a measure of risk.

The third objective of this book is to show that the perceived risk of loss sheds some light on investors’ behavior and on the dynamics of financial instability. It suggests in particular that a significant and protracted fall in the perceived risk of loss is a prerequisite for the inflation of major financial bubbles. Kindleberger claimed that some “good” news is the catalyst of a mania. The perceived risk of loss suggests an alternative explanation, namely, the persistent absence of “bad” news in the form of price drawdowns. To that extent, the perceived risk of loss is a concept to consider when discussing moral hazard.

Structure of This Book and Readers’ Guidance

This book is divided in four parts. Part I (chapter 1, Expectations Before the Rational Expectations Revolution, and chapter 2, Rational
Expectations) presents what belongs to common knowledge about expectations in economics. Its purpose is to facilitate an assessment of Allais’s contribution in this field. Readers who are novice at expectations theory but keenly aware that knowing the history of any discipline is key to understanding its current state of development will find this part more useful than experts in rational expectations.

Part II (chapters 3 to 6) introduces Allais’s theory of “expectations” under uncertainty in its original context, namely, macroeconomics and monetary dynamics. If there is one part in this book that even a hurried reader cannot afford to not read, it is this one. This is particularly true of chapter 4 (on the HRL formulation of the demand for money), which is really the heart of this book, since it expounds the theory of psychological time and of the time-varying rate of memory decay. Without knowledge of this chapter, it is not possible to benefit from reading any of the following ones. Readers interested in macroeconomics will find chapters 3 (Macrofoundations of Monetary Dynamics), 5 (The Fundamental Equation of Monetary Dynamics), and 6 (Joint Testing of the HRL Formulation of the Demand for Money and of the Fundamental Equation of Monetary Dynamics) more useful than those exclusively focused on behavioral economics and finance.

Part III (chapters 7 and 8) is essentially an invitation to extend the field of application of the theory of psychological time and memory decay beyond monetary dynamics. Any scholar or practitioner willing to test the HRL formulation in his or her own field of activity must read chapter 7, which illustrates the dynamic properties of the HRL formulation by means of a detailed numerical example based on a recent case of hyperinflation (Zimbabwe between 2000 and 2008). The reader making this effort will spare herself the misunderstandings or misinterpretations that have prevented some in the academic world, even from Chicago, from grasping the HRL formulation, when it was put forward by Allais.

In chapter 8, following Allais, we will extend the field of application of the theory of psychological time and memory decay to nominal interest rates. We will do so by looking at their correlation with the perceived rate of nominal growth in 18 countries, which have experienced over long periods very contrasted economic conditions ranging from deflation to hyperinflation. We will show that the theory of memory decay is also a theory of impatience, albeit not in the way Allais imagined.
This chapter may be of particular interest to readers concerned with intertemporal choice.

Part IV (chapters 9, 10, and 11) applies the HRL formulation to the analysis of financial instability thanks to the perceived returns and risks, which it derives from times series of returns on financial assets.

Chapter 9 presents a few simple models that explain financial behavior by the perceived returns on financial assets and thereby provide evidence of positive feedback from past returns to the demand for risky assets, a frequent conjecture that usually remains formulated in literary terms only. These models bring to light nonlinear relationships between perceived returns and observed investors’ behavior.

Chapter 10 highlights the psychological importance of downside risk by presenting Allais’s paradox and by contrasting how Allais and prospect theory have interpreted this paradox. Chapter 11 then explains how time can be introduced in the assessment of downside risk, that is, how the Allais paradox and his theory of psychological time can be combined into the perceived risk of loss. It illustrates the relevance of the perceived risk of loss by analyzing the pricing of some financial instruments. It suggests that major bubbles tend to be heralded by a fall in the perceived risk of loss to unprecedented levels. Finally, it discusses potential connections between the perceived risk of loss and moral hazard. These three chapters should interest any reader (scholar, practitioner, policymaker) eager to enhance the tools he or she uses to analyze financial market dynamics and to cope with financial instability.

The conclusion summarizes the key arguments in the form of a comparison of the HRL formulation with the REH.

As just said, the perceived risk of loss unifies two important contributions made by Maurice Allais: first, the well-known paradox that bears his name; second, his still rather confidential hereditary, relativist, and logistic (HRL) theory of economic and financial behavior under uncertainty. That these two contributions were made, respectively, almost 60 and 50 years ago is not a good reason for us to neglect them as outdated or irrelevant—quite the opposite. Often in the history of economic analysis, it has taken quite some time for important contributions to be fully recognized. Cantillon had to wait for Jevons to be recognized as an early master of monetary analysis. Gossen foreshadowed Jevons’s, Menger’s, and Walras’s marginal utility theory by almost three decades.
Admittedly, Allais was awarded the Nobel Prize in 1988. Yet, a large part of his works remains relatively unknown, if only because they were first published in French. Almost 30 years have elapsed since Paul Anthony Samuelson commented on Allais in the following terms:\textsuperscript{20} “Maurice Allais is a fountain of original and independent discoveries. Had Allais’ earliest writings been in English, a generation of economic theory would have taken a different course.”

This book will show that Samuelson’s statement certainly applies to the Allais theory of psychological time and time-varying rate of memory decay.

Any reader who is kind enough to send me (at ericbarthalon@gmail.com) a correction further debugging this book will receive a then-current complete errata sheet and many thanks. It goes without saying that the opinions expressed in this book are mine alone, and do not represent the views of my past employers or those of Allianz SE.

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### TABLE 1
Distinction Between Risk and Uncertainty

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<thead>
<tr>
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<th>Risk</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
<td>Existence and knowledge of a set of stationary frequencies computed by “the” model (the statistical parameters of this set do not change over time and their value is the same, irrespective of the estimation period)</td>
<td>Nonstationary frequencies (Keynes: “by very uncertain, I do not mean very improbable”)</td>
</tr>
<tr>
<td><strong>Probabilities</strong></td>
<td>Objective (all agents refer to the same set of frequencies)</td>
<td>Subjective (each agent is likely to have its own set of probabilities)</td>
</tr>
<tr>
<td><strong>Expectations</strong></td>
<td>Rational (people’s expectations are equal to the empirical frequencies)</td>
<td>Adaptive (the learning process takes time and is continuous)</td>
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