Notes

Introduction

1. Knight, F. (1921), *Risk, uncertainty and profit*, Beard Books, Washington, D.C., 2002, p. 19: "Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk. . . . The term 'risk,' as loosely used in everyday speech and in economic discussions, really covers two things which, functionally at least, in their causal relations to economic organization are categorically different. . . . The essential fact is that 'risk' means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. . . . It will appear that a measurable uncertainty, or 'risk' proper . . . is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term 'uncertainty' to cases of the non-quantitative type. It is 'true' uncertainty, and not risk . . . , which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition."

2. Lucas, R. E., Jr. (1977), *Understanding business cycles*, in Brunner, K., and Meltzer, A. (eds.), *Stabilization of the domestic and international economy*, vol. 5, Carnegie-Rochester Series on Public Policy, North Holland Publishing Company, New York, pp. 7–29.

3. Phelps, E. S., and Frydman, R. (2013), *Rethinking expectations: The way forward for macroeconomics*, Princeton University Press, Princeton, NJ.

4. Volatility is indeed volatile.

5. Guesnerie, R. (2001 and 2005), *Assessing rational expectations* (1 and 2), MIT Press, Cambridge, MA.

6. Kindleberger, C. P. (1978), *Manias, panics and crashes (A history of financial crises),* 4th ed., Wiley, New York, 2001.

7. In hedge finance, current cash flows cover debt servicing (capital and interest). In speculative finance, they only cover interest payments. In Ponzi finance, they cover neither interest nor principal payments.

8. As measured by a Fisher volume index eliminating currency fluctuations.

9. Caginalp, G., Porter, D., and Smith, V. L. (2000), *Overreactions, momentum, liquidity and price bubbles in laboratory and field asset markets*, The Journal of Psychology and Financial Markets, 2000, vol. 1, no. 1, pp. 24–48; Gjerstad, S., and Smith, V. L. (2009), *Monetary policy, credit extension, and housing bubbles: 2008 and 1929*, *critical review*, A Journal of Politics and Society, vol. 21, nos. 2–3.

10. Renshaw, E. (1988), *The crash of October 19 in retrospect*, The Market Chronicle, vol. 22, 1.

11. 1911–2010, 1988 laureate of the Prize in Economic Sciences in Memory of Alfred Nobel.

12. Fisher, I. (1896), *Appreciation and interest*, American Economic Association, Pickering & Chatto, London, 1996; Fisher, I. (1930), *The theory of interest*, Chap. 19, *Relation to money and prices*, Pickering & Chatto, London, 1996; Wicksell, K. (1898), *Interest and prices*, Chap. 8, *The natural rate of interest on capital and the rate of interest on loans*, Augustus M. Kelley, Fairfield, NJ, 1965; Knight, F. (1921), *Risk, uncertainty and profit*, Chap. 8, Beard Books, Washington, D.C., 2002; Hayek, F. (1936), *Economics and knowledge*, Presidential address delivered to the London Economic Club on November 10, 1936, Economica (1937), vol. 4, pp.33–54. Keynes, J. M. (1936), *The general theory of employment, interest and money*, Chap. 12, *The state of long-term expectations*, Harcourt Brace Jovanovich, New York, 1964; Hicks, J. (1939), *Value and capital*, Oxford University Press (repr.) 2001; Friedman, M. (1968), *Factors affecting the level of interest rates*, in *Savings and Residential Financing: 1968 Conference Proceedings*, Jacobs, D. P. and Pratt, R. T., (eds.), The United States Saving and Loan League, Chicago, IL, p. 375.

13. Allais, M. (1966), *A restatement of the quantity theory of money*, American Economic Review, vol. 56, no. 5, December, pp. 1123–1157.

14. Fisher, I. (1932), *Booms and depressions*, Pickering & Chatto, London, 1996; Fisher, I. (1933), *The debt-deflation theory of great depressions*, Econometrica, vol. 1, issue 4, October, pp. 337–357; Minsky, H. P. (1975), *John Maynard Keynes*, Columbia University Press, New York; Minsky, H. P. (1982), *Can "it" happen again?* Sharpe, New York; Minsky, H. P. (1986a), *The financial instability hypothesis: Capitalist production and the behavior of the economy in financial crises: Theory, history and policy,* eds. C. Kindleberger and J-P. Laffargue, Cambridge University Press, New York; Minsky, H. P. (1991), *The financial instability hypothesis: A clarification* in *The risk of economic crisis,* ed. Martin Feldstein; Kindleberger, C. P. (1985), *Rational expectations and*

collective memory, in *Keynesianism vs monetarism and other essays in financial history*, Allen and Unwin, London.

15. See, for example, Sargent, T. J. (1999), *The conquest of American inflation*, Princeton University Press, Princeton, NJ.

16. See appendix D.

17. Cagan, P. (1956), *The monetary dynamics of hyper-inflation*, in Friedman, M. (ed.), *Studies in the quantity theory of money*, University of Chicago Press, Chicago.

18. Camerer, C. F., Loewenstein, G., and Rabin, M. (2004), *Advances in behavioral economics*, Princeton University Press, Princeton, NJ; Allais, M. (1953), *Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école Américaine*, Econometrica, vol. 21, pp. 503–546; Allais, M. (1965), *Reformulation de la théorie quantitative de la monnaie*, Société d'études et de documentation économiques, industrielles et sociales (SEDEIS), Paris.

19. Cantillon, R. (1755), *Essai sur la nature du commerce en général*, Institut National d'Etudes Démographiques (INED), Paris, 1997.

20. Samuelson, P. A. (1982), *A chapter in the history of Ramsey's optimal feasible taxation and optimal public utility prices*, in *Economic essays in honour of Jorgen H. Gelting*, Andersen, S., Larsen, K., Norregard Rusmussen, P., and Vibe-Pedersen, J. (eds), Danish Economic Association, Copenhagen, pp. 164–165, note 3. Reprinted in the Collected Scientific Papers of P. A. Samuelson, vol. 5 (1986), MIT, Cambridge.

21. Keynes, J. M. (1936), *The general theory of employment, interest and money*, Chap. 12, *The state of long-term expectations*, Harcourt Brace Jovanovich, New York, 1964.

1. The Progressive Emergence of Expectations

1. Cantillon, R. (1755), *Essai sur la nature du commerce en général*, Institut National d'Etudes Démographiques, Paris, 1997, Part 1, Chap. 13.

2. Cantillon, *Essai sur la nature du commerce en général*, Part 2, Chap. 10.

3. In 1720, according to Cantillon, the market interest rate in London rose from 5 percent to 60 percent per annum.

4. Wicksell, K. (1898), *Interest and prices*, Chap. 8, *The natural rate of interest on capital and the rate of interest on loans*, Augustus M. Kelley, Fairfield, NJ, 1965.

5. The closeness of Wicksell's "gambling spirit" with Keynes's "animal spirits" is striking.

6. Wicksell, K. (1906), *Lectures on political economy*, vol. 2, Augustus M. Kelley, Fairfield, NJ, 1965.

7. And he adds the following: "Exactly the same effects would be visible with an unchanged, or even a higher, rate of interest, if meanwhile the expected profit on capital had considerably increased."

8. Hicks summarized this argument by saying it assumes the "elasticity of expectations" with respect to actual price changes to be equal to 1.

9. Knight, F. (1921), *Risk, uncertainty and profit*, Beard Books, Washington, D.C., 2002.

10. Keynes, J. M. (1936), *The general theory of employment, interest and money*, Chap. 12, *The state of long-term expectations*, Harcourt Brace Jovanovich, New York, 1964.

11. Much of his analysis of expectations was already present in Keynes, J. M. (1924), *A tract on monetary reform*, Prometheus Books, New York, 2000.

12. Schumpeter, J. (1939), *Business cycles.* Reprinted in 1989 by Porcupine Press, Philadelphia.

13. Haberler, G. (1937), *Prosperity and depression. A theoretical analysis of cyclical movements*, 2011, Transaction Publishers, New Brunswick, NJ.

14. Lundberg, E. (1937), *Studies in the theory of economic expansion*, Chap. 9, 1964, Augustus M. Kelley, Fairfield, NJ.

15. Hayek, F. (1936), *Economics and knowledge*, Presidential address delivered to the London Economic Club on November 10, 1936, Economica (1937), vol. 4, pp. 33-54.

16. Fisher, I. (1896), *Appreciation and interest*, American Economic Association, Pickering & Chatto, London, 1996.

17. Heterogeneity of forecasts, in modern parlance.

18. Fisher, I. (1930), *The theory of interest*, Chap. 19, *Relation to money and prices*, Pickering & Chatto, London, 1996.

19. Cagan, P. (1956), *The monetary dynamics of hyper-inflation,* in *Studies in the quantity theory of money*, ed. Milton Friedman, University of Chicago Press, Chicago.

20. Cagan approached the problem in terms of forecasting error and error correction coefficients, while Allais put the stress on the decay of human memory.

2. Rational Expectations Are Endogenous to and Abide by "the" Model

1. Muth, J. F. (1960), *Optimal properties of exponentially-weighted forecasts*, Journal of the American Statistical Association, vol. 55, no. 290, June, pp. 299–306.

2. Gourgieroux, C., and Monfort, A. (1995), *Séries temporelles et modèles dynamiques*, 2nd ed., Economica, Paris, pp. 106–110.

3. In chapter 3, we shall see that both Cagan and Allais empirically encountered and recognized the shortcomings of exponential smoothing in the early 1950s. It is this empirical encounter that led Allais to the HRL formulation.

4. Sargent, T. J., and Wallace, N. (1973), *Rational expectations and the dynamics of hyperinflation*, International Economic Review, vol. 14, no. 2, June.

5. Muth, J. F. (1961), *Rational expectations and the theory of price movements,* Econometrica, vol. 29, no. 3, July.

6. Lucas, R. E. (1972), *Expectations and the neutrality of money*, Journal of Economic Theory, vol. 4, pp. 103–124.

7. Lucas, R. E. (1976), *Econometric policy evaluation: A critique*, Carnegie-Rochester Conference Series on Public Policy, North Holland, New York, vol. 1, pp. 19–46.

8. Sargent, T. J., and Wallace, N. (1976), *Rational expectations and the theory of economic policy*, Journal of Monetary Economics, July, pp. 199–214.

9. Arrow, J. K. (1986), *Rationality of self and others in an economic system*, Journal of Business, vol. 59, no. 4, part 2, October, pp. 385–399, reprinted in *The New Palgrave: Utility and Probability*, Eatwell J., Milgate M., Newman P. (eds), Norton, New York.

10. Knight would have said *risky* instead of *uncertain*.

11. Malinvaud, E. (1991), *Voies de la recherche macroéconomique*, Points-Seuil, Paris, 1993, pp. 546–549.

12. Read *risk*.

13. Read *risk*.

14. Lucas, R., and Sargent, T. (1979), *After Keynesian macroeconomics*, The Federal Reserve Bank of Minneapolis, Quarterly Review 321, Spring.

15. Guesnerie, R. (2001), *Assessing rational expectations*, MIT Press, Cambridge, MA.

16. Modigliani, F. (1977), *The monetarist controversy, or should we foresake stabilization policy*? American Economic Review, vol. 67, no. 2, pp. 1–17, March.

17. Friedman, B. M. (1975), *Rational expectations are really adaptive after all*, unpublished paper, Harvard University, Cambridge, MA.

18. Lucas, R. (1975), *An equilibrium model of the business cycle*, Journal of Political Economy, vol. 83, no. 6, pp. 11–21.

19. Gowers, T. (ed.) (2008), *The Princeton companion to mathematics*, Princeton University Press, Princeton, NJ, p. 160.

20. In other words, $P(E) = \sum_i P(E|H_i) \times P(H_i)$ for all i.

21. Lindley, D. V. (1990), *Thomas Bayes* in *The New Palgrave: Utility and Probability*, Eatwell J., Milgate M., Newman P. (eds.), Norton, New York.

22. Parent, E., and Bernier, J. (2007), *Le raisonnement Bayésien: Modélisation et inférence*, Springer-Verlag, Paris.

23. Wonnacott, Thomas H., and Ronald, J. (1990), *Introductory statistics*, 5th ed., Wiley, New York.

24. Pastor, L., and Veronesi, P. (2009), *Learning in financial markets*, Annual Review of Financial Economics, vol. 1, pp. 361–381.

25. Sargent, T. J. (1999), *The Conquest of American inflation*, Princeton University Press, Princeton, NJ.

26. Another suggestion is to do rolling recursive OLS linear regression on an arbitrarily decided number of observations.

27. See, for example, Sargent, T. J. (1999), *The Conquest of American inflation*, Princeton University Press, Princeton, NJ.

28. www.nobelprize.org.

29. Kay, J. (2011), *The random shock that clinched a brave Nobel Prize*, Financial Times, October 18.

30. Gide, C., and Rist, C. (1944), *Histoire des doctrines économiques depuis les physiocrates jusqu'à nos jours*, Livre 6, Chap. 2, *Le conflit des théories des crises*, 6th ed., Dalloz, Paris, 2000.

31. Sargent, T., and Wallace, N. (1976), *Rational expectations and the theory of economic policy*, Studies in Monetary Economics, Federal Reserve Bank of Minneapolis, October 1976; Journal of Monetary Economics, July.

32. Phelps, E. (2009), *A fruitless clash of economic opposites*, Financial Times, November 3.

33. Arrow, J. K. (1986), *Rationality of self and others in an economic system*, Journal of Business, vol. 59, no. 4, part 2, October, pp. 385–399, reprinted in *The New Palgrave: Utility and Probability*, Eatwell J., Milgate M., Newman P. (eds.), Norton, New York.

34. Sargent, T. J., and Wallace, N. (1973), *Rational expectations and the dynamics of hyperinflation*, International Economic Review, vol. 14, no. 2, June.

35. Woodford, M. (1999), *Revolution and evolution in twentieth-century macroeconomics*, Princeton University Press, Princeton, NJ, note 51, p. 24.

36. Blaug, M. (1996), *Economic theory in retrospect*, 5th ed., Cambridge University Press, Cambridge, p. 685.

37. Screpanti, E., and Zamagni, S. (2005), *An outline of the history of economic thought*, Oxford University Press, Oxford.

38. Woodford, M. (2008), *Convergence in macroeconomics: Elements of the new synthesis*, prepared for the annual meeting of the American Economics Association.

39. The so-called Duhem-Quine problem.

40. Woodford, M. (2011), *What's wrong with economic models?* Institute for New Economic Thinking (INET), New York.

41. Lucas, R. (1995), *Monetary neutrality*, Nobel Prize lecture, December 7.

42. Bootle, R. (1997), *The death of inflation*, Nicholas Brealey, London. Quote used with kind permission of Roger Bootle.

43. Simon, H. A. (1997), *An empirically based microeconomics*, First lecture (Rationality in Decision Making), Raffaele Mattioli Foundation, Cambridge University Press, Cambridge.

Introduction to Part II

1. Allais, M. (2001), *Fondements de la dynamique monétaire*, Editions Clément Juglar, Paris.
2. Cagan, P. (1956), *The monetary dynamics of hyper-inflation*, in Friedman, M. (ed.), *Studies in the Quantity Theory of Money*, University of Chicago Press, Chicago.
3. Sargent, T. J., and Wallace, N. (1973), *Rational expectations and the dynamics of hyperinflation*, International Economic Review, vol. 14, no. 2.

3. Macrofoundations of Monetary Dynamics

1. Allais, M. (1954a), *Les fondements comptables de la macro-économique* , 2nd ed., Presses Universitaires de France, Paris, 1993.
2. In *Value and capital*, Hicks uses the same starting point, but—as far as I can judge—he did not exploit it as far as Allais did.
3. One simply needs to consider the government as an additional business selling services against taxes and to present the rest of the world as another additional business buying (importing) domestic goods and services or selling (exporting) foreign goods and services.
4. Blaug, M. (1996), *Economic theory in retrospect*, 5th ed., Cambridge University Press, Cambridge, Chap. 16, p. 646.
5. Blaug, M. (1996), *Economic theory in retrospect*, Chap. 16, p. 647.
6. Albeit arguably, fixed-term deposits are more likely to be precautionary balances than currency and demand deposits.
7. Blaug, M. (1996), *Economic theory in retrospect*, Chap. 5, pp. 54–55.
8. Gide, C., and Rist, C. (1944), *Histoire des doctrines économiques*, 6th ed., *Conflit des théories des crises*, Book 6, Chap. 2, Dalloz, Paris, 2000.
9. Allais, M. (1953b), *Illustration de la théorie des cycles économiques par un modèle monétaire non linéaire*. Extraits de la communication au Congrès Européen de la Société d'Econométrie, Innsbrück, 2/9/1953, Econometrica, January 1954, pp. 116–117.
10. Allais, M. (1954b), *Explication des cycles économiques par un modèle non linéaire à régulation retardée*. Communication au Congrès Européen de la Société d'Econométrie, Uppsala, 4/8/1954, Metroeconomica, vol. 8, fascicule I, pp. 4–83, April 1956.

11. In the Innsbruck paper, the two functions depend only on $x = \frac{1}{D}\frac{dD}{dt}$, the latest growth rate in nominal spending. In the two later papers, they depend on the exponentially smoothed sequence of growth rates.

12. This function was chosen for its simplicity and illustrative properties. It belongs to this book, not to Allais, but, needless to say, it is directly inspired by his later works.

4. Microfoundations of Monetary Dynamics

1. Allais, M. (1965), *Reformulation de la théorie quantitative de la monnaie*, Société d'études et de documentation économiques, industrielles et sociales (SEDEIS), Paris.

2. The two formulations become equivalent when the rate of inflation becomes dominant relative to the real rate of growth.

3. As in the Uppsala (1954) and Paris (1955) papers, Allais observes that this formulation is very close to Boltzmann's oblivion function and to Volterra's dampening function.

4. This recursive relationship makes it very easy to compute an exponential average.

5. Minsky, H. P. (1986b), *Stabilizing an unstable economy*, Yale University Press, New Haven, CT.

6. Quoted by the Financial Times, August 27 and 28, 2005.

7. In physical time.

8. In psychological time.

9. In physical time.

10. In psychological time.

11. Besides Einstein, there are many examples in literature and politics of implicit reference to psychological time. For example, Lenin is believed to have said, "Sometimes decades pass and nothing happens, and then sometimes weeks pass and decades happen."

12. The term Z is nothing but the numerator of relationship 4.2 under the assumption that r varies over time.

13. One of the fundamental principles of economic analysis is to present economic issues as optimization problems in which a certain quantity must be minimized or maximized. According to this principle, it seems logical to conjecture that relationship 4-31 (or 4-52) should be the solution of a certain optimization problem that, as it happened, Allais has not explicitly laid out. I wish I had been able to formulate this important problem before this book goes to press. Further research will hopefully close this gap.

14. For $E \approx 0$, $e^{-\alpha E-\gamma} \approx (1-\alpha E)e^{-\gamma}$.

15. Allais, M. (2001), *Fondements de la dynamique monétaire*, pp. 82–84 and pp. 469–472.

16. If $\alpha = 0$ or $b = 0$, then $\Psi(Z) = 1$ and the HRL formulation becomes equivalent to an exponential average, where χ_0 remains the only parameter.

17. Caginalp, G., Porter, D., and Smith, V. L. (2000), *Overreactions, momentum, liquidity and price bubbles in laboratory and field asset markets*, The Journal of Psychology and Financial Markets, vol. 1, no. 1, pp. 24–48.

18. Prat, G., (1999), *Temps psychologique, oubli et intérêt chez Maurice Allais, Discussion Papers (REL–Recherches Economiques de Louvain)* 65(2) 1999022, Université catholique de Louvain, Institut de Recherches Economiques et Sociales (IRES), Louvain.

19. Portelli, G., Ruffier, F., Roubieu, F. and Franceschini, N. (2011), *Honeybees' speed depends on dorsal as well as lateral, ventral and frontal optic flows*, Plos ONE, May, vol. 6, Issue 5.

5. The Fundamental Equation of Monetary Dynamics

1. Allais, M. (1968), *L'équation fondamentale de la dynamique monétaire*, Monnaie et Développement, fascicule 1, Ecole Nationale Supérieure des Mines de Paris, pp. 75–86.

2. In the Anglo-Saxon literature, transactions are usually denoted by the letter T; however, as Allais uses the letter T to denote his response period, it seems appropriate to designate transactions with the letter Q to prevent any risk of confusion.

3. As opposed to the income velocity of money v, which is defined by the ratio $v = \Upsilon/M$, where Υ represents national income.

4. Leaving aside, for the time being, nonbank credit.

5. Hicks, J. (1939), Value and capital Part III, *The foundations of dynamic economics*, Chap. 9, *The method of analysis*, reprinted in 2001 by Oxford University Press, Oxford.

6. He even states that a first-rate business should show flexibility in this regard.

7. Allais, M. (1968b), *La génération endogène des fluctuations conjoncturelles*, reprinted in *Fondements de la dynamique monétaire,* Editions Clément Juglar, Paris, 2001, pp. 969–1010.

8. Samuelson's oscillator has the same mathematical form, but its theoretical foundations, variables, and parameters are totally different.

9. Allais, M. (1982), *La génération exogène des fluctuations conjoncturelles*, Rapport d'activité scientifique du Centre d'Analyse Economique, CNRS, pp. 17–27, July.

10. Allais, M. (1983), *Fréquence, probabilité et hasard*, CNRS, Journal de la Société Statistique de Paris.

6. Joint Testing of the HRL Formulation of the Demand for Money and of the Fundamental Equation of Monetary Dynamics

1. Notes 10–13, pp. 1256, 1258, and 1259.

2. Friedman M., Jacobson Schwartz A., (1963), *A monetary history of the United States 1867-1960*, Princeton University Press, Princeton, NJ.

3. See, for example, Quantitative Micro Software, (2007) *EViews 6, User's guide*, Irvine, CA or the works of the Nobel laureates Software who 'formulated these techniques: Engle', Granger, Sims.

4. For example, the GARCH(1,1) specification

$$Y_t = X_t' + \epsilon_t \tag{1}$$

$$\sigma_t = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

is equivalent to exponentially weighted moving average (EWMA) variance measures defined by the recursion

$$\sigma_t^2 = (1-\lambda)\epsilon_t^2 + \lambda\sigma_{t-1}^2 \tag{3}$$

where $\omega = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$.

7. Allais's HRL Formulation: Illustration of Its Dynamic Properties by an Example of Hyperinflation (Zimbabwe 2000–2008)

1. Allais, M. (1966), *A restatement of the quantity theory of money*, American Economic Review, vol. 56, no. 5, pp. 1123–1156, December.

2. Cagan, P. (1964), *Comment by Phillip Cagan*, in *Fondements de la dynamique monétaire*, Editions Clément Juglar, Paris, pp. 1057–1060; Cagan, P. (1969), *Allais' monetary theory, interpretation and comment*, Journal of Money, Credit and Banking, pp. 427–462, August; Darby, M. (1970), *Allais' restatement of the quantity theory: Comment*, American Economic Review, pp. 444–446, June; Scadding, J. L. (1972), *Allais' restatement of the quantity theory of money: Note*, American Economic Review, pp. 151–154, March.

3. Allais, M. (1969), *Growth and inflation, A reply to the observations of the discussants*, Journal of Money, Credit and Banking, pp. 427–462, August; Allais, M. (1970), *Allais' restatement of the quantity theory: Reply*, The American Economic Review, pp. 447–456, June; Allais, M. (1975), *The hereditary and relativistic formulation of the demand for money, circular reasoning or a real structural relation? A reply to Scadding's note*, American Economic Review,

pp. 454–464, June; Allais, M. (1986b), *The empirical approaches of the hereditary and relativistic theory of the demand for money: Results, interpretation, criticisms and rejoinders*, Economia della Scelte Pubbliche, Journal of Public Finance and Public Choice (Fondazione Luigi Einaudi), pp. 3–83.

4. Allais used an IBM 7094 computer.

5. Allais, M. (1972), *Forgetfulness and interest*, Journal of Money, Credit and Banking, pp. 40–73, see note 22, pp. 46–47, February.

6. J. P. P. Henri (or Henry) (1848–1907).

7. Case in which the constant is forced to zero, that is, when the average forecasting error is assumed to be equal to zero.

8. See section 2.12.

9. Kindleberger, C. (1985), *Keynesianism vs. monetarism and other essays in financial history*, Chap. 6, *Collective memory vs. rational expectations: Some historical puzzles in macro-economic behavior*, Allen & Unwin, London.

10. In his *Economics of inflation*, a book whose subject is the German hyperinflation, Bresciani-Turoni speaks most of the time of the depreciation of the mark. His original title in Italian is *Le vicende del Marco Tedesco*. The first sentence of Lionel Robbins's foreword to the English edition starts with the words "The depreciation of the mark."

11. Hanke, S. H., Krus, N., (2012), World hyperinflations, Cato Institute Working Paper no 8, August 15.

12. Shiller, R. J. (1999), *Irrational exuberance*, Princeton University Press, Princeton, NJ.

13. This is in sharp contrast with Andrei Shleifer's formulation where the reaction of positive feedback traders is supposed to be proportional to the latest price change. See Shleifer, A. (2000), *Inefficient markets (An introduction to behavioral finance)*, Oxford University Press, Oxford.

8. The HRL Formulation and Nominal Interest Rates

1. Allais, M. (1972), *Forgetfulness and Interest*, Journal of Money, Credit and Banking, February; Allais, M. (1974), *The psychological rate of interest*, Journal of Money, Credit and Banking, August.

2. During a conversation on April 25, 2002, Maurice Allais bluntly told the author of this book that his theory of the psychological rate of interest "did not work."

3. Allais, M. (1943), *A la recherche d'une discipline économique*, Clément Juglar, Paris, 1994, Allais, M. (1947), *Economie et intérêt*, Clément Juglar, Paris, 1998.

4. O'Donoghue, J., Goulding, L., and Allen, G. (2004), *Consumer price inflation since 1750*, Economic Trends, no. 604, pp. 38–46.

5. Polanyi, K. (1944), *The great transformation*, Gallimard, Paris, 1972.

6. Datastream time series: USGDP . . . B.

7. Bresciani-Turoni, C. (1937), *The economics of inflation*, Augustus M. Kelley, New York.

8. Homer, S., and Sylla, R. (1996), *A history of interest rates*, 3rd ed., Rutgers University Press, New Brunswick, NJ.

9. Allais, M. (1960), *Influence du coefficient capitalistique sur le revenu réel par tête*, 32nd Congress of the International Statistical Institute, Tokyo, Document no 61, June; Allais, M. (1962), *The influence of the capital-output ratio on real national income*, Econometrica, October, vol. 30, pp. 700–728; Desrousseaux, J. (1961), *Expansion stable et taux d'intérêt optimal*, Annales des Mines, November, pp. 829–844; Phelps, E. S. (1961), *The golden rule of accumulation: A fable for growthmen*, The American Economic Review, vol. 51, September, pp. 638–643; Robinson, J. (1962), *A neoclassical theorem*, Review of Economic Studies, June, vol. 29, pp. 219–226; von Weizsäcker, C. (1962), *Wachstum, Zins und optimale Investisionsquote*, J. C. B Mohr (Paul Siebeck), Tübingen; and Swan, T. (1963), *Of golden ages and production functions*, in Berril, K. (ed.), *Economic development with special reference to East Asia*, St. Martin's, New York: we refer the reader to their work for a detailed demonstration of the golden rule of accumulation.

10. Cantillon, R. (1755), *Essai sur la nature du commerce en général*, INED, Paris, 1997; or Mill, J. S. (1848), *Principles of political economy*, Augustus M. Kelley, Fairfield, NJ, 1987.

11. Gibson, A. H., (1923), *The future course of high class investment values*, Banker's Magazine (London), 115, January, pp. 15–34.

9. Perceived Returns and the Modeling of Financial Behavior

1. Speculative or Ponzi finance to borrow Minsky's words.

2. Akerlof, G. A., and Shiller, R. J. (2009), *Animal spirits*, Princeton University Press, Princeton, NJ.

3. Datastream time series USCBDMGNA.

4. NBER Macro History database time series M14074: US loans on securities by member banks.

5. NBER Macro History database time series M12017a: Bank debits NYC USD bn.

6. NBER Macro History database time series M13003: NYSE 90-day time loan.

7. See for example the famous statement made by Chuck Prince, then Citigroup's CEO, in an interview to the Financial Times on July 9, 2007: "When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing".

8. See table 4.5, page 89.

9. Greenspan, A. (2002), *Economic volatility*, Remarks at the Symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 30, Federal Reserve Board, Washington, D.C.

10. Downside Potential Under Risk: The Allais Paradox and Its Conflicting Interpretations

1. Koulovatianos, C., and Wieland, V. (2011), *Asset pricing under rational learning about rare disasters*, Institute for Monetary and Financial Stability, Goethe-Universität, Frankfurt am Main, Working Papers Series no. 46.

2. As 1 ducat weighs about 0.11 troy ounce, assuming a gold price of USD 1,300, one ducat is worth about 143 current USD in early 2014.

3. Bernoulli, D. (1954), *Exposition of a new theory on the measurement of risk,* Econometrica, vol. 22, no. 1, pp. 26–36. Translated from Latin into English from Papers of the Imperial Academy of Science in Petersburg, vol. 6, 1738, pp. 175–192.

4. Kahneman, D. (2011), *Thinking, fast and slow*, Chap. 25, *Bernoulli's errors*, Farrar, Straus and Giroux, New York.

5. In many problems, ordinal utility suffices to find solutions.

6. Allais, M. (1943), *A la recherche d'une discipline économique*, Clement Juglar, no. 69, pp. 162–165.

7. J. K. Arrow, M. Boiteux, D. van Dantzig, B. de Finetti, M. Fréchet, M. Friedman, R. Frisch, R. Gibrat, G. Guilbaud, R. Hutter, W. Jaffé, H. Lavaill, J. Marschak, E. Malinvaud, P. Massé, G. Morlat, R. Roy, P. A. Samuelson, L. J. Savage, L. J. Shackle, J. Ville, and H. Wold.

8. Allais, M. (1953), *La psychologie de l'homme rationnel devant le risque: la théorie et l'expérience*, Journal de la Société Statistique de Paris, Book 94, pp. 47–73; http://archive.numdam.org/ARCHIVE/JSFS/JSFS 1953 94 /JSFS 1953_1953 94 47 0.pdf; in its full version, the questionnaire consisted of 400 questions.

9. Questions 35 and 36, the only two questions reproduced in Allais's ubiquitously quoted 1953 Econometrica article.

10. 100 million 1952 FFR is roughly equivalent to 2.536 million 2012 USD.

11. Using 1 million as unit.

12. Without referring to utility, it is possible to give a less rigorous but may be more intuitive explanation of the Allais paradox. Respondents should have preferred B to A and D to C, since B and D have higher mathematical expectation than, respectively, A and C. Yet, in choice 1, a majority of respondents preferred A to B, while in choice 2, a majority of the same respondents preferred D to C. The preference $D \succ C$ is consistent with the maximization of

mathematical expectation, since D is the prospect with the higher probability-weighted gain (500m versus 110m). In contrast, the preference $A \succ B$ contradicts the maximization of mathematical expectation, since A has lower mathematical expectation than B(100m versus 139m). In terms of mathematical expectation, the difference between A and C is the same as between B and D, namely, 89m = 89% × 100m. The empirical observation that A is preferred to B and C to D suggests that the outcome (100m; 89%), which is embedded in both A and B, is not taken into account independently from the other outcomes (x_i, p_i) present in these two prospects, contrary to the independence axiom. In other words, the outcome (100m; 89%) seems to be valued more in A, where there is no risk at all, than in B, which entails some risk.

13. Kahneman, D., and Tversky, A. (1979), *Prospect theory: An analysis of decision under risk*, Econometrica, vol. 47, no. 2, March.

14. It is not clear whether prospect theory considers gains and losses in absolute or relative terms, although the first hypothesis seems the most likely.

15. Kahneman, D. (2011), *Thinking, fast and slow*, Chap. 26, *Prospect theory*, Farrar, Straus and Giroux, New York.

16. See problems P_3 in table 10.4 and P_3' in table 10.5.

17. See Kahneman and Tversky, *Prospect theory: An analysis of decision under risk*, Econometrica, vol. 47, no. 2, March p. 269.

18. Kahneman, D., and Tversky, A. (1992), *Advances in prospect theory: Cumulative representation of uncertainty*, Journal of Risk and Uncertainty, vol. 5, pp. 297–323.

19. In the first two problems, the values of the gains and losses are close enough to ignore the potential impact of utility. This approach is more questionable in the last two problems, even though local linearity of cardinal utility may still be assumed.

20. The *possibility effect* alluded to by Kahneman and Tversky can seemingly be measured by odd-order moments.

21. In these two problems, the gains are so close to each other that the potential impact of utility can safely be ignored.

22. Here, too, the potential impact of utility can probably be ignored as the values of the gains are in a maximum ratio of 2 to 1. The same remark applies to table 10.5.

23. The *certainty effect* alluded to by Kahneman and Tversky can seemingly be measured by even-order moments.

24. Rabin, M., and Thaler, R. (2001), *Anomalies: Risk aversion*, The Journal of Economic Perspectives, vol. 15, no. 1, pp. 219–232.

25. Allais, M. (1979) [1952], *The foundations of a positive theory of choice involving risk and a criticism of the postulates and axioms of the American school*, in Allais, M., and Hagen, O., *Expected utility and the Allais paradox*, D. Reidel, Dordrecht, pp. 24–145.

26. Before we proceed further, let us observe that an empirical observation of the general type

$$\bar{s}(U_4) - \bar{s}(U_3) = \bar{s}(U_2) - \bar{s}(U_1) \tag{1}$$

implies that cardinal utility can only be determined up to a linear transformation. Let us assume that cardinal utility is actually a linear transformation f of $\bar{s}$, such that

$$f(U) = c_1\bar{s}(U) + c_2 \tag{2}$$

Then if empirical observation reveals that relative to U_3, the gain U_4 provides the same increase in satisfaction as U_2 with respect to U_1, the only relationship we can write is

$$f(U_4) - f(U_3) = f(U_2) - f(U_1) \tag{3}$$

which is equivalent to

$$c_1\bar{s}(U_4) + c_2 - c_1\bar{s}(U_3) - c_2 = c_1\bar{s}(U_2) + c_2 - c_1\bar{s}(U_1) - c_2 \tag{4}$$

and finally to

$$\bar{s}(U_4) - \bar{s}(U_3) = \bar{s}(U_2) - s(U_1) \tag{5}$$

The information contained in c_1 and c_2 is lost.

27. The answers given by the subject (Finetti in this case) are underlined; in his questionnaire, Allais formulated his questions using round numbers of 1952 French francs, such as 10, 25, and 50 million. To facilitate a contemporary reader's introspection, these round numbers have been converted to 2012 USD and rounded to the nearest $1,000, at the risk of some persistent oddity.

28. In contrast, Kahneman and Tversky (1979) only allude to minimum perceptible thresholds.

29. Allais, M. (1977), *The so-called Allais' paradox and rational decisions under uncertainty*, in Allais, M., and Hagen, O. (1979), *Expected utility hypotheses and the Allais paradox.*

30. Allais, M. (1943), *A la recherche d'une discipline économique*, paragraph nos. 68–69, pp. 156–177.

31. $d^2\bar{s}/dU^2 < 0$ and $d^3\bar{s}/dU^3 < 0$

32. In a lin-log graph, one plots the abscissa along a base-10 logarithmic scale and the ordinates along a linear scale.

33. Bear in mind that this analytical work was conducted some years before the advent of Excel, MATLAB, or Eviews as we know them today.

34. Allais, M. (1986a), *Determination of cardinal utility according to an intrinsic invariant model*, in *Recent developments in the foundations of utility and risk theory*, D. Reidel Publishing Company, Dordrecht, pp. 83–120.

35. Negligible as it may seem, this small hump provides some interesting insights as regards hyperbolic discounting (see appendix E on intertemporal choice).

36. Allais, M. (1991), *Cardinal utility: History, empirical findings and applications. Theory and decision 31*, Kluwer Academic, Dordrecht, pp. 99–140.

37. Questions 71 to 78 and 90 to 98.

38. 1952 French francs.

39. And not *below.*

40. Allais, M. (1977), *The so-called Allais' paradox and rational decisions under uncertainty*, in Allais and Hagen, (1979), *Expected utility hypotheses and the Allais paradox*, pp. 481–3, 550–52, 607–9.

41. Mehra, R., Prescott, E.C. (1985), *The equity premium: A puzzle*, Journal of Monetary Economics, 15, pp. 145–161.

42. Benartzi, S., Thaler, R. (1995), *Myopic loss aversion and the equity premium puzzle*, Quarterly Journal of Economics, vol. 110, no. 1, February, pp. 73–92.

43. Data source: Morningstar, (2014), *2014 Ibbotson Stocks, Bonds, Bills and Inflation (SSBI) Classic Yearbook*, Morningstar, Chicago, IL.

11. Downside Potential Under Uncertainty: The Perceived Risk of Loss

1. From a purely formal point of view, there is no difference at all between mathematical expectation and a cocktail recipe. But someone ordering a gin and tonic is not expecting to be served two full glasses of gin first, followed by five full glasses of tonic (or vice versa).

2. To take an extreme example, until Monday, October 19, 1987, close of business, nobody knew that the Dow Jones Index could fall by 22.6 percent in one trading session.

3. Marks, H. (2011), *How quickly they forget*, Letter to Oaktree's clients.

4. By relative spread, we mean the difference between the logarithms of BAA and AAA bond yields.

5. Median price of existing homes.

6. A simple way to visualize the role of α is to observe the behavior of the logarithm of the rate of memory decay when the present value of past returns tends toward infinity. From the definition of $\chi(Z)$, we have

$\lim_{Z \to +\infty} \ln \chi(Z) = \alpha Z + \ln \chi_0 + \ln b - \ln(1 + b)$, which is the equation of an asymptotic line having α for slope.

7. With

$$\chi_b(Z) = \chi_0 \frac{1 + be^{\alpha Z}}{1 + b} \tag{1}$$

and

$$\chi_1(Z) = \chi_0 \frac{1 + be^{\alpha Z}}{2} \tag{2}$$

we have indeed

$$\chi_b(Z) = \chi_1(Z) = \frac{\chi_0}{2}(e^{\alpha Z} - 1)(b - 1) \tag{3}$$

8. Keynes's famous analogy between financial markets and beauty contests could be given a quantitative content by assuming $\alpha > 1$ and $b < 1$, or $\alpha' > 1$ and $b' > 1$ or both!

9. This assumption concurs with recent research. See, for example, Malmendier, U., and Nagel, S. (2009), *Depression babies: Do macroeconomic experiences affect risk-taking?* and (2011), *Learning from inflation experience*, UC Berkeley and Stanford University, NBER and CPER. The two authors claim to have found that "individuals learn from data experienced over their life-times, rather than from all 'available' data." For example, "young individuals place more weight on recently experienced inflation than older individuals," or "individuals who have experienced low stock-market returns throughout their lives report lower willingness to take financial risk" and "recent return experiences have stronger effects, but experiences early in life still have significant influence, even several decades later."

12. Conclusion

1. Pribram, K. (1951), *Prolegomena to a history of economic reasoning*, The Quarterly Journal of Economics, vol. 65, no. 1, February.

2. Phelps, E. (1985), *Political economy: An introductory text*, Norton, New York.

3. Phelps, E. (1987), *Marchés spéculatifs et anticipations rationnelles*, Revue Française d'économie, vol. 2, no. 3.

4. Allais, M. (1989), *Autoportraits, ma philosophie de la méthode économique*, Montchrestien, Paris.

Appendix C Proofs

1. Rinne, H. (2003), *Taschenbuch der Statistik*, Harri Deutsch Verlag, Frankfurt am Main, pp. 213–217.

2. Bronstein, I. N., Semendjajew, K. A., Musiol, G., Mühlig, H., (2006), *Taschenbuch der Mathematik*, 6th ed., Verlag Harri Deutsch, Frankfurt am Main, p. 461.

3. The subscript e designates the equilibrium value of all the variables considered in this demonstration.

4. Neglecting second-order terms, we have, for example,

$$\lim_{g\to 0} \frac{\Psi(Z_e+g)-\Psi(Z_e)}{g} = \Psi'(Z_e) \Rightarrow \Psi(Z) = \Psi(Z_e+g) \approx \Psi(Z_e) + g\Psi'(Z_e) \tag{1}$$

$$\Psi(Z) \approx \Psi(Z_e) + g\Psi'(Z_e) \tag{2}$$

$$\frac{\Psi|(Z)}{\Psi(Z_e)} \approx \frac{\Psi(Z_e)+g\Psi'(Z_e)}{\Psi(Z_e)} \approx 1 + \frac{\Psi'(Z_e)}{\Psi(Z_e)} g \approx 1 - K_e g \tag{3}$$

$$\frac{\Psi(Z)}{\Psi(Z_e)} \approx 1 - K_e g \tag{4}$$

$$V(t) = V_e(1+f(t)) \Rightarrow \ln V(t) = \ln V_e + \ln(1+f(t)) \tag{5a}$$

$$\Rightarrow \frac{1}{V}\frac{dV}{dt} = \frac{d\ln V(t)}{dt} = \frac{d\ln(1+f(t))}{dt} \approx \frac{df(t)}{dt} \tag{5b}$$

$$\frac{1}{V}\frac{dV}{dt} \approx \frac{df(t)}{dt} \tag{6}$$

Appendix E A Note on the Theory of Intertemporal Choice

1. Camerer, C., Loewenstein, G., and Rabin, M. (2004), *Advances in behavioral economics*, Princeton University Press, Princeton, NJ.

2. Samuelson, P. A. (1937), *A note on measurement of utility*, Review of Economic Studies, vol. 4, pp. 155–161.

3. Frederick, S., Loewenstein, G., and O'Donoghue, T. (2002), *Time discounting and time preference: A critical review*, Journal of Economic Literature, vol. 40, pp. 351–401, June.

4. Discount factors are the present value–future value ratios of outcomes deemed equivalent albeit distant in time.

5. See Table 6-1 in Frederick, S., Loewenstein, G., and O'Donoghue, T., *Time discounting and time preference.*

6. See chapter 8 of this book.

7. See Frederick, S., Loewenstein, G., and O'Donoghue, T., *Time discounting and time preference.*

8. A possibility alluded to by Daniel Bernoulli in 1738, quoting a contribution by Gabriel Cramer, dated 1728.

9. Thaler, R. (1981), *Some empirical evidence on dynamic inconsistency*, Economic Letters, vol. 8, pp. 201–207.

10. Please note that these rates are annual continuous (log) rates.

11. Smoothed by means of Excel's built-in function.

12. Relationship E.7 is identical to relationship 4.37, section 4.3, p. 76.