Resizing Risks

Scanning price charts for channels has a bad reputation among economic theorists. Good traders do it anyway for the insight into uncertainty. It’s a secret of their success that shouldn’t stay secret. The extra information can warn of crisis and help all investors dynamically resize their portfolios. However, it will never completely tame market risk. Pandora’s Equation won’t let it.

While VaR lost every tournament described in Chapter 10, conventional standard deviation estimators didn’t win. Estimators based on daily trading range walloped both. This chapter will explain how they work and show some useful things we can do with them.

Range estimators are a formalization of what good traders have done for generations. Uncertainty is center stage for traders, and their livelihoods depend on managing it right, unlike a host of other risk managers who just report measures to others. So it makes sense that traders would evolve useful, intuitively appealing techniques.

In saying that, I have violated one of the norms of finance. It says that theorists and discretionary traders should never take each other seriously. The very existence of this norm invites a closer look.

Traders as Chartists

Any big bookstore in a major financial district will have a large section on financial engineering. Some of the books there expound standard finance
theory. Many more instruct in so-called technical analysis, also known as charting.

This is strange, very strange. No major university offers comprehensive professional instruction in market technical analysis. One never hears of tenure-track appointments for chartists, much less Nobel prizes awarded for their profound insight. Lo and Hasanhodzic (2009, 2010), two scholar-practitioners trying to bridge the gap, are welcome exceptions. Why do traders demand so much instruction in something academics rarely provide?

To some this just confirms traders’ irrationality. Still, the scale and intensity amaze. Bookstores near pharmaceutical labs don’t sell alchemy recipes. Bookstores near NASA don’t sell incantations for magic carpets.

Traders’ unusual reading predilections merit more scholarly attention. If some Amazon tribes were discovered to eat bat guano, ethnobiologists would leap to explore the benefits. But I have never seen a medical study of the therapeutic effects of charting.

If anthropologists study professional traders in their habitats, one thing they will immediately notice is the close quarters. Big trading outfits tend to cram desks and people back-to-back and side-to-side. The few offices tend to be walled in glass, the better to see out and look in. Traders also tend to cluster in compact financial districts, despite the ease of relocating somewhere cheaper and more serene. Everything is geared to maximize face-to-face communication.

Indeed, great discretionary traders tend to be first and foremost great networkers. They are far more interested in market beliefs than in economic fundamentals. Their knowledge is broad rather than deep, intuitive rather than analytic, and short on detail.

That doesn’t mean they’re stupid. Computers often have trouble recognizing faces that two-year-olds can distinguish at a glance. Why? Human brains are wired with millions of lateral connections, whereas computers are wired more serially. Great discretionary traders combine the intuitive pattern recognition of a child with a grasp of abstract market connections. In a sense they read the faces of the market.

Nowhere are those faces more visible than in the charts of market prices. Yes, the charts are backward-looking and noisy. Still, no other record better summarizes the hopes and disappointments that underlie current beliefs.

Readers schooled in orthodox finance may doubt whether that’s necessary. The parameters describing drift and volatility ought to tell
us everything of note. I agree. So let’s go back to the bookstore and pick up the latest reference handbook on diffusion parameters of various assets.

Oh no, it isn’t there. Amazon.com doesn’t carry it either. Yet it is easy to find reference handbooks on diffusion parameters for basic industrial materials.

Perhaps financial diffusion coefficients are too valuable to publish. Perhaps they’re stored in encrypted files at major banks and hedge funds. Then why hasn’t some hacker found them and published them online? Why hasn’t some disgruntled quant kissed and told? Moreover, if the true values are already known, why do quants spend so much of their time re-estimating them?

The only plausible answer is that the values aren’t stable. We likely never knew them precisely and never will. Studying price charts is a search for relevant approximation.

### Adjusted Trading Range

For a simple charting analysis, graph a time series of (log) prices, pick an interval without a glaring break in trend, and draw a straight line from the first point to the last. Then draw two parallel lines with the same slope, just touching the outliers on either side. All prices in that interval are confined to the channel in between.

Figure 11.1 presents a simulation. I generated random variables from a $t$-distribution of kurtosis of 6 and formed the cumulative sum to represent a log price series. Although there’s no inherent drift, the observed trend slopes upward.

I will call the vertical distance between the two channel lines the adjusted range. It tends to scale directly with the cumulative volatility. (The cumulative volatility is the square root of the cumulative variance, which equals the average variance times the time interval.) Hence, we can use adjusted range to back out a point estimate for volatility. We can then combine these point estimates into a range-based estimator by taking moving averages or dynamic mixtures of moving averages.

In principle, range-based estimators of volatility are far more precise than ordinary standard deviation estimators. To confirm that, let’s start with a little transformation. If we subtract the observed trend from the sample, prices will start and end at the same value. The adjusted trading
range will stay the same but now equals the ordinary high-minus-low range for the adjusted series. See Figure 11.2.

In the case of Brownian motion, the adjusted series is known as a Brownian bridge. “Brownian” refers to a continuous-time random walk with no discrete jumps. The “bridge” refers to it being tied down at both ends rather than one. For a \( t \)-distribution, there’s no formal name, partly because continuous time muddies the very notion of a \( t \)-distribution; we need to shift to Brownian motion with occasional discrete jumps.
The mean range of a Brownian bridge works out to twice the mean absolute change, or $\sqrt{\pi/2} \approx 1.25$ times the cumulative volatility. The standard relative error of an average of $T$ i.i.d. samples works out to

$$SRE(\text{Brownian Bridge Range}) \equiv \frac{0.217}{\sqrt{T}}, \quad (11.1)$$

which is less than a third of the SRE for an ordinary $T$-period standard deviation estimator. To put it another way, the width of the current day’s trading channel can, in principle, estimate volatility better than ten days of closing prices, even when the regime stays the same.

**Practical Approximations**

A range measure filters a larger data set for the difference between two extremes. For more precision we can incorporate finer data directly into an ordinary variance calculation. For any Brownian displacement $\Delta x$, if we measure the displacements over $n$ subintervals and add their squares, this estimates variance $n$ times as precisely as $(\Delta x)^2$ does. By subdividing the subdivisions, we can in principle measure current volatility to arbitrary precision in infinitesimal time.

In practice, precision at short time scales is limited. Brownian approximations break down. Bid-ask spreads distort our measures. We lack sufficiently fine-grained data or run out of storage capacity for it.

Adjusted trading range offers a good proxy. On intervals of a day or longer it typically dwarfs big-ask spreads. The human eye (more precisely, the brain using visual data as input) can quickly identify the main patterns and exceptions.

When we don’t have enough information to calculate the adjusted trading range directly, we can estimate it using logarithmic high/low/open/close (HLOC) data. A first approximation is

$$\text{adjRange} \equiv \text{high} - \text{low} - \frac{1}{2} \cdot |\text{close} - \text{open}|, \quad (11.2)$$

which is the standard range less half the absolute return. The Appendix explains the intuition and makes some refinements to improve the projections of volatility.

To illustrate the application, let’s return to the SPX example from the previous chapter, using HLOC data from Yahoo Finance. Highs and lows
are available from 1962 onward; opens were recorded until recently as the previous day’s close. Figure 11.3 depicts the (log) volatility calculated from two-week and one-year averages of range-based measures. Both averages are centered as best possible. In practice we wouldn’t obtain the full year of data until six months after the date we’re pinning it to.

If risks were stable, the one-year average line should look relatively flat, typically within 10% (0.1 log unit) of its mean. Instead it swings by more than a factor of 6 (1.8 log units). Eighteen months in 2007 and 2008 sufficed to traverse most of that range. Yet volatility can be range-bound for years at a time or shift in a relatively smooth trend. There are also fluctuations of a few months’ duration that are too big to dismiss as random noise and too irregular to treat as a fixed cycle.

Volatility does seem to revert to mean over a five-year horizon. However, it is not clear the mean is stable. Indeed, the chart suggests a rising long-term trend, although our vision may be distorted by the crisis near the end and the limited liquidity near the beginning.

At the very least, the chart challenges the presumption that advanced technology, lower transaction costs, and better educated traders must inexorably grind volatility down. While those developments do bring prices closer to consensus, they also make it easier to trade on fine differences in
opinion and small nuggets of information. A sophisticated market need not be a calm one.

Indeed, judging from Figure 11.3 alone, it seems hard to predict anything about volatility other than its relative continuity and eventual pullback from extremes. Hence, most of what we call volatility forecasting is simply trying to track it closely and project a short-term continuation or reversion. That’s more the rule than the exception in finance.

Two range-based estimators are charted in Figure 11.4 for 2007 through 2010, along with the average range-based volatility looking two weeks ahead. An EMA with an average lag of two weeks significantly outperforms an EMA with an average lag of three months. In September 2008 it is an excellent barometer of crisis, as it indicates volatility has doubled. In 2009 it is an excellent barometer of tensions easing. Dynamic mixtures can provide similar early warnings with fewer fluctuations during calm.

**Why Is Volatility Volatile?**

Financial econometricians have for a generation recognized that the volatility is volatile. Engle (1982) developed tractable models under the name Autoregressive Conditional Heteroskedasticity, or ARCH. Bollerslev (1986)
introduced a Generalized ARCH or GARCH model, which has spun out numerous variations.

The basic idea of GARCH is to treat the variance $\sigma^2$ of a random walk as following a random walk itself. To keep $\sigma^2$ from wandering off to infinity or down below zero, GARCH models its drift as mean-reverting and its volatility as a power function of $\sigma$. A basic specification in continuous time is

$$d\sigma^2 = a(1 - b\sigma^2)dt + c\sigma^m dz,$$

(11.3)

where $a$, $b$, $c$, and $m$ are positive constants and $dz$ represents standardized Brownian motion with zero drift and unit rate of variance. Relationships like these are known as GARCH processes.

In most GARCH processes $m = 1$, although Heston (1993) developed a popular option pricing model in which $m = \frac{1}{2}$. For $m = 1$, equation (11.3) matches equation (8.1), with $\sigma^2$ replacing the default risk $\theta$. It follows that $\sigma^2$ will have a long-term equilibrium gamma distribution. This suggests modeling tail risks with a normal-gamma distribution. However, that’s not nearly as tractable as the $t$-distribution, which implicitly assumes the inverse variances are gamma distributed. Moreover, the differences are mild, as the logarithm of a gamma-distributed variable is approximately normally distributed, and the logarithm of the inverse variable will have the same distribution with the signs reversed. In either case, financial market tail risks will decay log-linearly or slower.

Empirically, GARCH models work like mixtures of EMAs. However, unlike the dynamic mixtures I recommend, most researchers look for a single best fit for the period as a whole. Indeed, many researchers speak of GARCH processes as if they are stable drivers of volatility and not just convenient estimators. This is misleading in theory and dangerous in practice.

To start with the practical danger, let’s return to the historical chart of range-based SPX volatility and cover up the last two years. Suppose we estimate fixed GARCH parameters for the previous 46 years. No matter how sophisticated the model we choose, the best fit will almost surely breed false confidence in avoiding the volatility experienced in fall 2008. While dynamic estimators may be no less myopic, at least they will adjust quickly to new evidence. Also, we can inject more caution by keeping tabs on past episodes of false confidence.

On the theory side, GARCH is less a causal explanation than a descriptor looking for a cause. If the price-to-dividend ratio stays constant or
changes only gradually, one might attribute GARCH behavior in prices to an underlying GARCH process in dividends. However, this just shifts the puzzle somewhere else. Why should dividend noise behave like GARCH?

In fact GARCH behavior demonstrates the power of beliefs. Recall our discussion in Chapter 7 of beliefs about default risk. They are inherently unpredictable, because the change in the $m^{th}$ cumulant $\kappa_m$ of beliefs depends on the next higher-order cumulant $\kappa_{m+1}$. For beliefs about log dividends, the analogous result is even more striking:

$$\text{volatility}(\kappa_m) = \frac{|\kappa_{m+1}|}{\text{volatility (dividends)}}. \quad (11.4)$$

Not surprisingly, the $m^{th}$ cumulant of prices turns out to move with the $m^{th}$ cumulant of beliefs about dividends. It follows that the variance of market prices will be volatile except in the rare case when beliefs are completely unskewed.

To confirm that completely unskewed beliefs are rare, note that skewness will be volatile except when kurtosis is zero, and more generally that $\kappa_m$ cannot stay zero unless $\kappa_{m+1}$ stays zero. The only way volatility can stay nonvolatile is for beliefs about drift to stay perfectly Gaussian. In practice that’s impossible.

In short, we don’t need to assert the volatility of price volatility. Learning under uncertainty implies it. However, the controlling parameters will hardly ever be constant. Neither can we completely predict their evolution.

Rational Exuberance

It’s not just the volatility of volatility that reflects rational learning. The average level of volatility does too. This merits more discussion, as much of behavioral finance clings to the notion that markets are excessively volatile.

The notion dates to Shiller (1981), who compared the volatility of SPX with the volatility of dividends accruing to SPX. With a constant price-to-dividend ratio, the volatility of log prices should match the volatility of log dividends. With a price-to-dividend ratio that discounts temporary shocks, the volatility of log prices should be less than the volatility of log dividends. Shiller found that, on the contrary, market prices were much more volatile than dividends.
This finding sparked enormous debate. Orthodox finance inclined to minimize the importance of the aberration. Behavioral finance took it as proof of gross investor irrationality. Shiller (2006) characterizes the problem as “irrational exuberance.” That is, emotion causes investors to exaggerate the importance of minor news. Taleb (2004) attributes the exaggeration to investors being “fooled by randomness.”

Each of us needs only review his own life to realize that emotion or foolishness often prevails. However, experience also confirms the adage about fools getting parted from money. From an evolutionary perspective, it is hard to understand why a rational minority does not come to dominate the market.

In fact, as Kurz (1994a, 1994b, 1996) first showed, rational learning is bound to induce excess volatility. If a dividend is better than expected, it may be a sign we weren’t expecting enough. Perhaps the firm’s growth prospects are improving. Perhaps some bad luck before caused us to underestimate growth prospects. Rationally updating our beliefs using Bayes’ Rule, we’ll expect slightly more growth going forward than we would if the dividend were poor. With higher growth, we’re willing to pay a higher price-to-dividend ratio.

The next dividend may be unexpectedly poor. We then lower our expectations and the price-to-dividend ratio we will offer. If we imagine news about dividends arriving continuously, its noisiness will constantly jiggle the price-to-dividend ratio.

Hence, market price volatility will sum two components. The first is the volatility of dividends. The second is the volatility of the price-to-dividend ratio caused by learning. Shiller’s analysis ignored the second component.

How much extra volatility will learning induce? That depends on what we believe and how intensely we believe it. From equation (11.4), the volatility of the mean belief depends on the variance of beliefs. The less certain we are about growth, or the more we disagree with each other, the more the consensus will shift with new evidence.

The price impact, in turn, depends on how long we expect the new growth trend to last. If growth is expected to revert to mean within a month, even huge disagreements will have miniscule impact. If growth trends are expected to persist for decades, the volatility when we’re highly uncertain will dwarf the news itself. The observed patterns of excess volatility are broadly consistent with economic cycles lasting a few years.

In hindsight, this can look quite irrational. We’ll see a few long trends and a lot of confused wandering around them. The same news will evoke vastly different responses, depending on the context. Yet high volatility
isn’t necessarily unhealthy. It just means that the market feels perplexed about the future and is eagerly sifting new evidence for insight.

Risk-Adjusted Investment

Enlightenment is the flip side of disillusionment. So if the difficulty of predicting market volatility discourages, look at the bright side. Let it encourage more attention to tracking current volatility and to adjusting investment appropriately.

For independent investments, the best single adjustment applies the fraction Kelly criterion mentioned in the previous chapter. It says to bet in proportion to expected net-of-funding return \( \mu \) divided by variance \( \sigma^2 \), or equivalently to the Sharpe ratio \( S \equiv \frac{\mu}{\sigma} \) divided by the volatility. For example, if volatility doubles, investors should look to trim their investment unless \( \mu \) quadruples or \( S \) doubles.

Of course, the practical application hinges on keeping transaction costs under control. One simple approach sets a wide target range stretching across the ideal holding. It rebalances only when the actual holding falls outside the target range, and trades only enough to bring the holding back to the edge of the range. More sophisticated variants set inner and outer target ranges with fractional trading in between. We also need to bear in mind the tax impact of frequent adjustments.

Nevertheless, the swings in market volatility are so extreme that it seems cruel not to alert retail investors to them. Financial investors rightly encourage retail investors to focus on the long run and to diversify their holdings. Mutual funds and unit trusts arose to supply this exposure. In recent years, exchange-traded funds and some EU-regulated funds have delivered more exotic risk-reward exposures. Intermediaries need to take the next step and offer more clearly risk-adjusted exposures.

It is easy to convert any liquid stock index into a volatility-adjusted index. Just construct a predictor of next-day volatility using information at hand, adjust the what-if holding of the index to aim for unit volatility, record the result, and repeat. Initially the predictor might not be very good, as we’ve seen with VaR. But we can make it better by folding range measures, implied volatilities, and information from other assets into dynamic mixtures. Given enough practice and publicity, some investment advisors may take enough comfort to use volatility-adjusted indices as benchmarks for their own performance.
That’s where the fun begins. The network externalities that favor a few leading currencies or credit ratings will also encourage markets to focus on a few volatility-adjusted benchmarks. Given enough appeal, brokers will offer financial derivatives mimicking their performance. That will reduce transaction costs and broaden their appeal.

This is bound to make markets function better. For starters, I see evidence that volatility smoothing improves Sharpe ratios long-term. To confirm this requires analysis of dozens of major equity indices around the world, along with adjustments for funding costs, dividends, and taxes. To avoid short-circuiting the deeper treatment this deserves, let me just offer this as a conjecture.

If testing confirms the conjecture, it represents a major market anomaly. While in principle everyone could take advantage of the anomaly, large-scale attempts to do so might wipe it out. Moreover, having retail investors trim equity holdings when volatility surges and expand when volatility recedes could aggravate market fluctuations. There’s no way these innovations can yield all gain and no pain.

Still, even if long-term equilibrium leaves average Sharpe ratios unchanged, the world as a whole stands to benefit. Many hedge funds serving the ultra-rich already manage their volatility closely. Assisting retail investors in doing so would help level the playing field. By tempering sharp drawdown in crisis, volatility smoothing would also help enlist broader participation in financial markets at less personal risk.

For those who fear that market fluctuations get worse, please bear in mind that—

- The key to trimming market swings lies in dampening the credit cycle, not in slowing investor response.
- If our savings and regulatory institutions can’t let social imagination play, let’s redesign them with sturdier buffers.
- Early warning of market uncertainty may help policymakers avert market panic.

“You’re right, Prometheus. Trading range indicates volatility much better than the net return does. With finer precision, no one can doubt that volatility is volatile. It shows the crucial importance of tracking changes short-term.”
“There is some longer-term mean reversion as well,” said Prometheus. “I’m glad Osband used your equation to explain the combination. Too many theorists simply invoke GARCH as a deus ex machina.”

“Speaking of gods, that excess volatility reminds me of the climb up Mt. Olympus,” said Pandora. “Men didn’t know the way. They searched and got lost. Zeus had plenty of time to gather lightning bolts and strike them down.”

“What does that have to do with excess volatility?”

“The best path always looks straighter in hindsight. Looking back, analysts see things that past markets could not.”

“They’ll know more if they study the path itself. Chartists have done that for generations. Economists should pay more attention. Perhaps they are irrationally under-exuberant.”

“Perhaps they are,” said Pandora. “However, I don’t blame them for having their guard up. Chartists speak a lot of gobbledygook, while traders exaggerate their prowess. It’s going to take a lot of work to mend the divide.”


“Let’s not understate the progress. Finance theory embeds risk at its core. It didn’t always. Mean-variance analysis of portfolios dates to the 1950s. Analytic formulas for derivative prices date to the 1970s.”

“Finance theory still tends to assume the risks are generally known. They aren’t. They can’t be. Even the beliefs can’t be completely known. Your equation proves it.”

“Yes, financial risk and uncertainty are twins. Finance theory needs to embed both at its core, along with the learning that describes their interaction.”